Predicting critical crashes? A new restriction for the free variables.

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1 Introduction

Several authors have noticed the signature of log-periodic oscillations prior to large stock market crashes [1], [2], [3]. Unfortunately good fits of the corresponding equation to stock market prices are also observed in quiet times. To refine the method several approaches have been suggested:

- Logarithmic Divergence: Regard the limit where the critical exponent β converges to 0. [3]
- Universality: Define typical ranges for the free parameters, by observing the best fit for historic crashes. [4]

We suggest a new approach. From the observation that the hazard-rate in [4] has to be a positive number, we get an inequality among the free variables of the equation for stock-market prices.

Checking 88 years of Dow-Jones-Data for best fits, we find that 25% of those that satisfy our inequality, occur less than one year before a crash. We compare this with other methods of crash prediction, i.p. the universality method of Johansen et al., which followed by a crash only in 9% of the cases.

Combining the two approaches we obtain a method whose predictions are followed by crashes in 54% of the cases.

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2 The hazard rate

In [4] Johansen et al suggest, that during a speculative bubble the crash hazard rate $h(t)$, i.e, the probability per unit time that the crash will happen in the next instant if it has not happened yet, can be modeled by by

$$h(t) \approx B_0(t_c - t)^{-\alpha} + B_1(t_c - t)^{-\alpha} \cos(\omega \log(t_c - t) + \psi).$$

By assuming that the evolution of the price during a speculative bubble satisfies the martingale (no free lunch) condition, they obtain a differential equation for the price $p(t)$ whose solution is

$$\log\left(\frac{p(t)}{p(t_0)}\right) = \kappa \int_{t_0}^{t} h(t')dt'$$

before the crash. Here κ denotes the expected size of the crash.

This implies that the evolution of the logarithm of the price before the crash and before the critical date t_c is given by:

$$(*) \quad \log(p(t)) \approx p_c - \frac{\kappa}{\beta} B_0(t_c - t)^{\beta} - \frac{\kappa}{\sqrt{\beta^2 + \omega^2}} B_1(t_c - t)^{\beta} \cos(\omega \log(t_c - t) + \phi)$$

With $\beta = 1 - \alpha$, p_c the price at the critical date, and ϕ a different phase constant.

Now the hazard rate is a probability and therefore positive. This leads to a necessary condition:

$$\begin{aligned} & 0 \le h(t) \\ \iff \quad & 0 \le B_0(t_c - t)^{-\alpha} + B_1(t_c - t)^{-\alpha} \cos(\omega \log(t_c - t) + \psi) \\ \iff \quad & 0 \le B_0 + B_1 \cos(\omega \log(t_c - t) + \psi) \end{aligned}$$

since $t < t_c$. At some times near the critical date $\cos(\omega \log(t_c - t) + \psi)$ takes on the values -1 and 1. This implies the necessary conditions

$$0 \le B_0 \pm B_1 \iff |B_1| \le B_0.$$

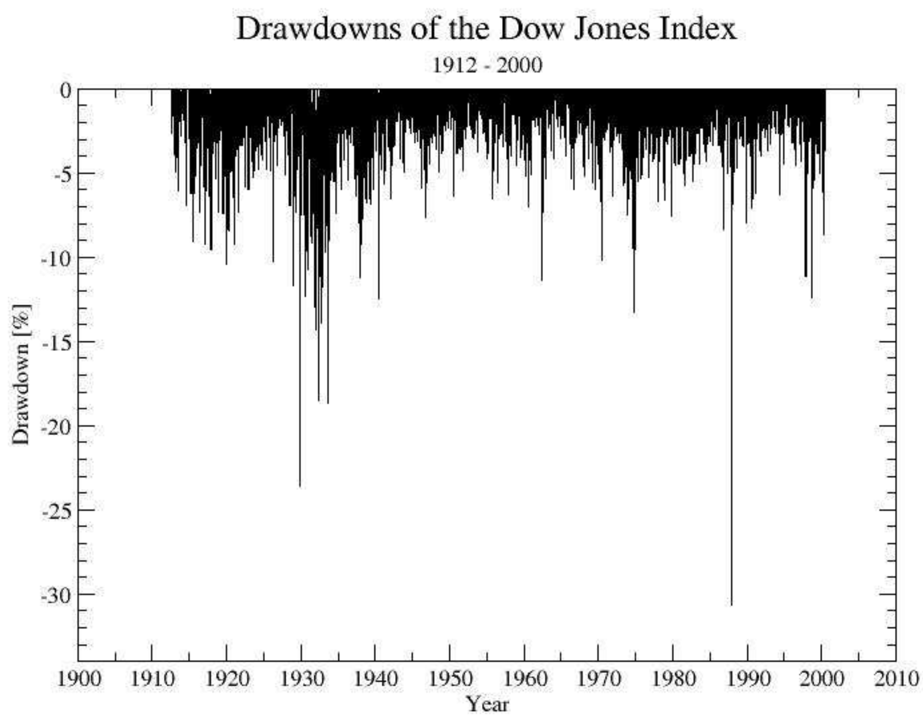
On the other hand these conditions are also sufficient for $h(t) \ge 0$ since $\cos(\omega \log(t_c - t) + \psi)$ is always between -1 and 1.

To summarize, if the assumptions of Johansen et al are valid, we must have $|B_1| \le B_0$ prior to a critical crash.

3 88 Years of Dow Jones

To check this model of speculative bubbles, we have investigated the Dow Jones index from 1912 to 2000. This period contains 23668 trading days.

Johansen et al define a crash as a continuous drawdown (several consecutive days of negative index performance) larger than 15%. The following diagram shows the drawdowns of the Dow Jones index from 1912 to 2000. Observe that there have been 4 drawdowns larger than 15% namely in the years 1929, 1932, 1933 and 1987.



As our basic data set, we have calculated numerically the best fit of equation (∗) to a sliding window of 750 trading days, every 5 trading days. This yields 4761 best fits. The complete data set is available at http://btm8x5.mat.uni-bayreuth.de/~bothmer .

In what follows, we will call a crash prediction successful, if it was issued at most one year before a crash. With this definition there are 229 best fits that could possibly give a successful crash prediction. By predicting crashes randomly, one would obtain a successful prediction in 4.8% of the cases.

3.1 Mean square errors χ

The first approach to detecting speculative bubbles is to look for good fits of $(*)$ to the Dow Jones Index. If the mean square error χ of the fit is sufficiently small one issues a crash prediction.

The next figure shows the mean square errors of all our best fits compared with the best fits before a crash. Unfortunately small errors also occur in quiet times.

If one issues a crash prediction if the mean square error is smaller then 0.75 one obtains:

	before crash	not before crash
no crash prediction ($\chi \geq 0.75$)	175	2799
crash prediction ($\chi < 0.75$)	72	1732

I.e. only $72/(72 + 1732) \approx 3.9\%$ of the predictions are successfull. Since this is worse than issuing random predictions, we conclude that one can not

predict a crash by looking only at the mean square error. This observation has also been made by Sornette and Johansen.

3.2 Critical Times t_c

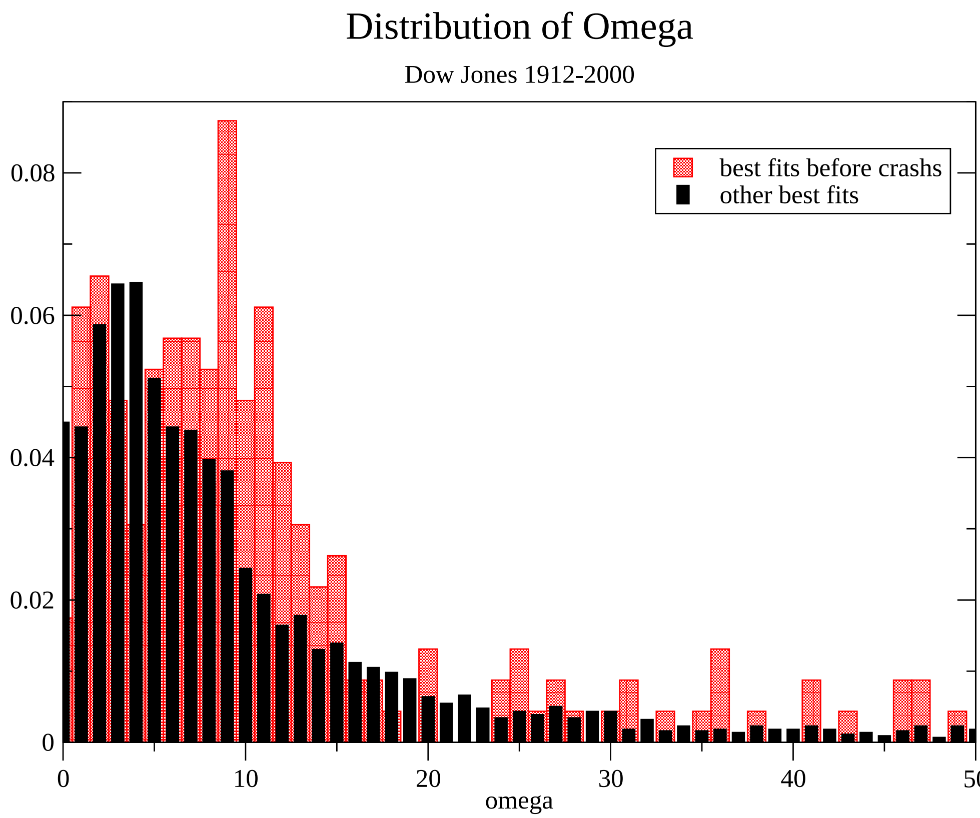
If the model of Johansen et al is correct one should expect that a crash occurs close to the critical date t_c. Using this one can issue a crash prediction when the critical date t_c is less than one year away. Using our dataset this lead to:

	before crash	not before crash
no crash prediction ($t_c \geq today + 1\,year$)	93	2652
crash prediction ($t_c < today + 1\,year$)	136	1879

I.e 136/(136+1879) $\approx$ 6.7% of the predictions are successful. This is slightly better that random predictions, but still not very good.

3.3 Universality

Johansen et al suggest that speculative bubbles exhibit universal behavior. This would imply that β and ω take on roughly the same values for each speculative bubble. The following diagram shows the distribution of ω before crashes compared with the distribution during other times.

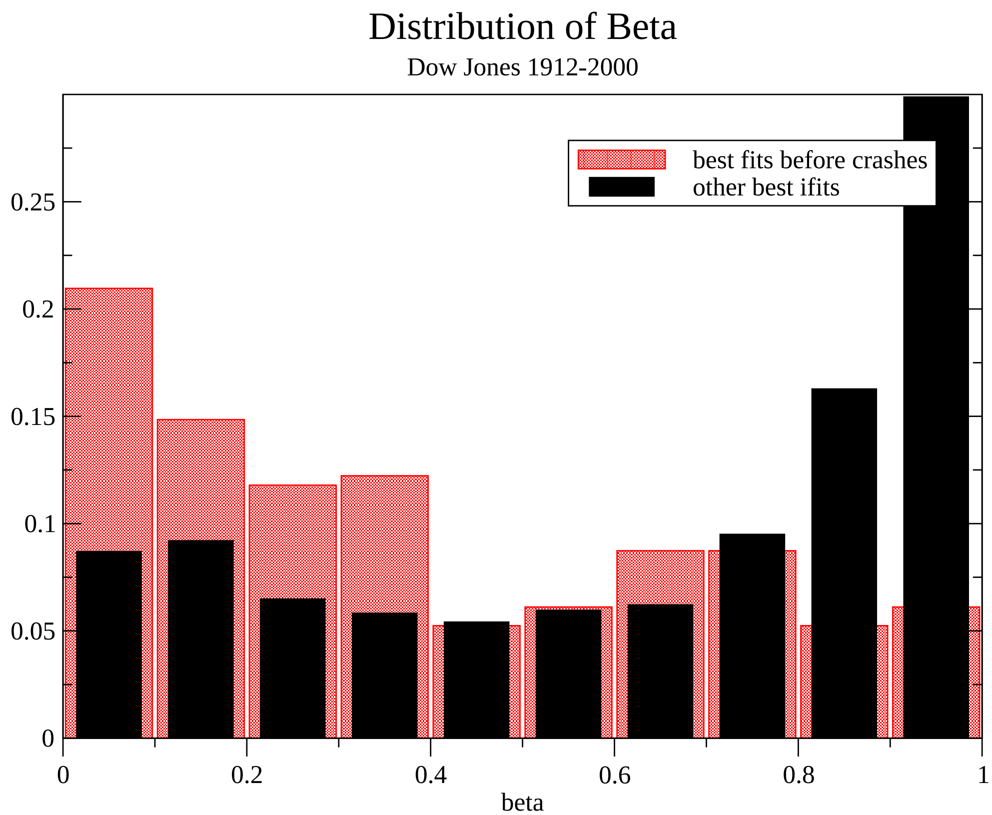


One can clearly observe an unexpected peak around $\omega = 9$ before the crashes. If we issue a crash prediction in the range $7 < \omega < 13$ we obtain:

	before crash	not before crash
no crash prediction	150	3728
crash prediction ($7 < \omega < 13$)	79	803

I.e $79/(79 + 803) \approx 8.9\%$ of the predictions are successful.

The distribution of β before crashes and not before crashes is:



Here we observe a tendency toward lower values of β, but no clear peak. We interpret this as evidence, that one should look for logarithmic divergence, i.e. the limit of β tending to 0, as suggested by Vandewalle et al. [3]. We will investigate this approach in a later paper.

3.4 Positive hazard rate

In section 2 we have explained that in the model of Johansen et al. the hazard rate $h(t)$ must be positive. We proved that this is equivalent to

$$|B_1| \leq B_0.$$

From our best fits we can calculate the value

$$\kappa(B_0 - |B_1|),$$

which should also be positive during a speculative bubble, since κ is a positive number. Consequently we can issue a crash warning if $\kappa(B_0 - |B_1|)$ is positive. With our dataset we obtain:

	before crash	not before crash
no crash prediction ($\kappa(B_0 - \lvert B_1 \rvert) \leq 0$)	133	4255
crash prediction ($\kappa(B_0 - \lvert B_1 \rvert) > 0$)	96	276

I.e $96/(96 + 276) \approx 25,8\%$ of the predictions are successful. This is already a practical success rate, but we can do even better, if we combine this with universality.

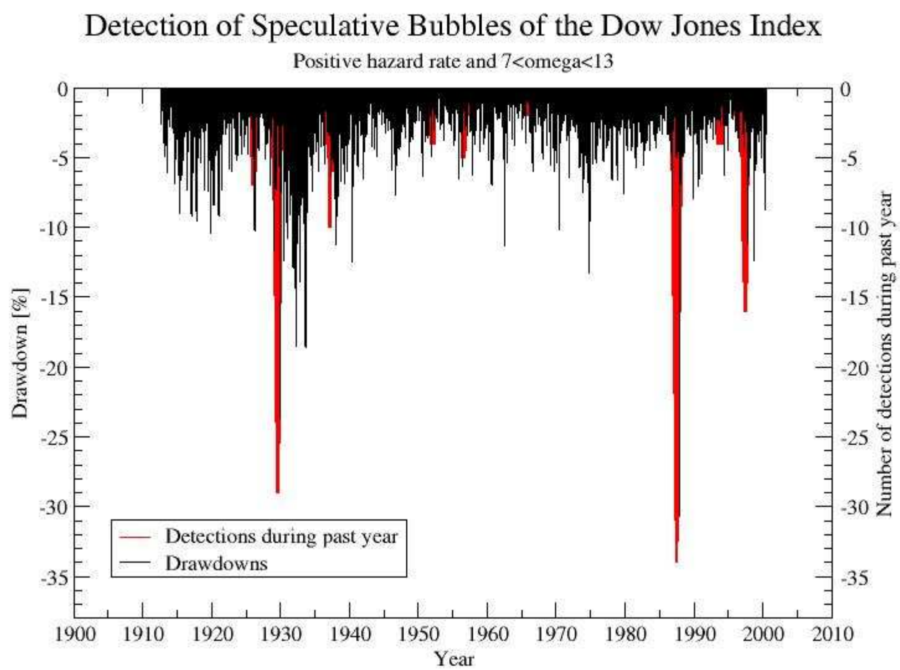
3.5 Positive hazard rate and universality

Combining the last two approaches we issue a crash prediction, if the hazard rate is everywhere positive and ω is in the range of section 3.3. This gives

	before crash	not before crash
no crash prediction	164	4476
crash prediction ($\kappa(B_0 - \lvert B_1 \rvert) > 0$ and $7 < \omega < 13$)	65	55

I.e $65/(65 + 55) \approx 54.1\%$ of the crash predictions are successfull.

The following diagram shows when these crash predictions where issued. For every trading day we have plotted the number of crash predictions during the past year and the drawdown of the Dow Jones index.



Notice that the crashes of 1929 and 1987 have been predicted well in advance. The crashes of 1932 and 1933 have not been directly predicted, but we argue that they are in the aftermath of 1929 and represent the bursting of the same speculative bubble. The crash predictions of 1997 where followed by two small crashes in October 1997 and 1998, which didn't quite reach 15%. One could argue that they represent a crash in two steps.

4 Summary

We have derived a new restriction of the free variables in the model of Johansen et al [4] for stock market prices during a speculative bubble. This restriction alone yields crash predictions with a 25% successrate for the Dow Jones index. This is an improvement over the 9% successrate obtained by using universality. Combining our approach and the universality method we obtain a success rate of 54%.

We think that these results represent strong evidence for the model of Johansen et al describing speculative bubbles in the stock market.

References

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